New ordering Approach (cont.)

\[ t_U = \beta + n_{UT} \ln(L_{R-n_{UT}}) \]
\[ t_L = \beta - n_{UT} \ln(L_{N-n_{UT}}) \]

\( n_{UT} \) is number of sensors who did not yet transmit

largest possible magnitude contribution from the sum of the sensor LLRs that did not yet transmit

Once sum of LLRs from transmitting sensors is larger than \( t_U \), sum will have to be larger than \( \beta \), regardless of the data at the sensors that didn't transmit.

Thus even without transmitting further, we can implement the energy unconstrained approach, with fewer transmissions.

So with Prob one we save energy, but How large are Gains from Ordering?

For any reasonable N-sensor binary hypothesis problem (\( P_e \) decreases with SNR-like quantity \( s \)), for sufficiently large \( s \) the average number of transmissions saved over the optimum unconstrained energy approach is strictly larger than \( N/2 \).

Example: Known signal in additive noise with symmetric unimodal noise pdf

\[ x_j = \theta + n_j \]
\[ \begin{cases} 
\theta = 0 & \text{under } H_0 \\
\theta = s > 0 & \text{under } H_1 
\end{cases} \]

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• Average number of transmissions saved over the optimum unconstrained energy approach for signal strength $s = 0.8$, various number of sensors and prior probabilities $\pi_1 = \text{Prob}(H_1)$, and a known signal in $\mathcal{N}(0,1)$ Gaussian noise.

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• Average number of transmissions saved over the optimum unconstrained energy approach for signal strength $s = 0.8$ and various number of sensors for a known signal in $\mathcal{N}(0,1)$ Gaussian noise.

Rick S. Blum, ECE Dept. Lehigh University
Ordering for Signal Detection


Outline

MIMO Radar
- Network of radars concept
- Diversity and resolution gains
MIMO Radar is Attracting Attention

- Research groups from all over the world

People working on MIMO Radar
MIMO Radar Special Issue -2009

Two types of MIMO Radar

- Classified by antenna configurations
The Radar MIMO Concept

- With MIMO radar, many "independent" radars collaborate to average out target fluctuations, while maintaining the ability to detect target (measure range, AOA).

- MIMO radar offers the potential for significant gains:
  - Detection/estimation performance through diversity gain
  - Resolution performance through spatial resolution gain (create bandwidth)

- A first step to a cooperative radar network.

Motivation

- Conventional radars experience target fluctuations of 5-35 dB

- Slow RCS fluctuations (Swerling I model) cause long fades in target RCS, degrading radar performance.

- MIMO radar exploits the angular spread of the target backscatter in a variety of ways to extend the radar’s performance envelope.

Backscatter as a function of azimuth angle, 10-cm wavelength [Skolnik 2003].
Traditional Beamformer

- A typical array radar is composed of many closely spaced antennas.
  - The performance of radar systems is limited by target RCS fluctuations, which are considered as unavoidable loss.
  - The common belief is that radar systems should maximize the received signal energy. This is made possible by the high correlation between the signals at the array elements.

MIMO with Angular Diversity

Angular spread - a measure of the variability of the signals received across the array. A high angular spread implies low correlation between target backscatter

MIMO mode:
- Exploit angular spread
- Orthogonal waveform

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Signal Model

- Point source assumption dominates current models used in radar theory.
- This model is not adequate for an array of sensors with large spacing between array elements.
- Distributed target model

![Diagram of many random scatterers]

Signal Model (Cont.)

\[ \mathbf{r}(t) = \sqrt{\frac{E}{M}} \mathbf{H} \mathbf{s}(t - \tau) + \mathbf{n}(t) \]

- Signal energy: \( E \)
- Transmit antennas: \( M \)
- Vector of transmitted waveforms: \( \mathbf{s}(t) \)
- Observation: \( \mathbf{r}(t) = [r_1(t), \ldots, r_M(t)]' \)
- Channel matrix: target gain between transmitter \( k \) and receiver \( l \):
  \[ [\mathbf{H}]_{lk} = h_{lk} \]
- Noise:
  \[ \mathbf{n}(t) = [n_1(t), \ldots, n_M(t)]' \sim \mathcal{CN}(0, \sigma^2) \]

Distributed Target

- It can be shown that \( h_{jk} \sim CN(0,1) \)
- Take \( h_{jk} \) and \( h_{il} \). We can show that if antenna not in single beamwidth then \( E\{h_{jk}h_{il}^H\} \approx 0 \), otherwise \( E\{h_{jk}h_{il}^H\} \approx 1 \).

Radar Detection Problem

- The radar detection problem:
  \( \text{Hyp}_0 \) : Target does not exist at point in space
  \( \text{Hyp}_1 \) : Target exists at point in space

- Assume that all the parameters are known. The optimal detector is the LRT detector, and it is given by

  \[ T = \log \frac{P(\mathbf{r}(t) | \text{Hyp}_1)}{P(\mathbf{r}(t) | \text{Hyp}_0)} > \delta \]

- The channel matrix \( \mathbf{H} \) is unknown, but its distribution is assumed known

  \[ P(\mathbf{r}(t) | \text{Hyp}_1) = \int P(\mathbf{r}(t) | \mathbf{H}, \text{Hyp}_1) P(\mathbf{H}) d\mathbf{H} \]

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Detection with MIMO Radar

- It can be shown that the sufficient statistic for the MIMO radar problem with unknown channel is the vector $x$, the output of a bank of matched filters sampled at $\tau$.
- Orthogonality of waveform is assumed
- The optimal NP detector is
  \[ T = \begin{cases} \frac{\|x\|^2}{\delta} & H_0 \\ \frac{\|x\|^2}{\delta} & H_1 \end{cases} \]
  where \( \delta = \frac{\sigma^2}{2} F^{-1}_\text{PNL} \left( 1 - P_{FA} \right) \)
- Non-coherent processing in a network of $MN$ radars. UMP Test, invariant extensions…

Prob(Miss) to illustrate Diversity Gain

- MIMO diversity (blue) Beamforming (red)

Miss prob versus SNR (fixed false alarm prob $10^{-6}$)
Resolution Gain: Coherent Processing Example

- The presented plots are based on a $L \times L$ system with transmitter and receiver elements equally distributed over $[-45^\circ, 45^\circ]$.

Resolution Gain: Coherent Processing Example

- MSE of ML estimate
- Ambiguity function (mainlobe width on order of wavelength)

MSE $\propto \lambda^2$ (wavelength$^2$) for high SNR

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Resolution Gain: Coherent Processing Example

- Increasing $MN$ gives lower sidelobes

Resolution Gain: Coherent Processing Example

- Increasing $MN$ gives lower sidelobes and better MSE
Questions?

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