A New Scheme for Energy-efficient Estimation in a Sensor Network

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***Abstract*—In this paper, energy efficient estimation of an unknown parameter in Gaussian noise is studied in a sensor networking context. A new approach is suggested to obtain a good approximation to the traditional maximum likelihood (ML) estimate, which can save energy by reducing the number of sensor transmissions. Specifically, we describe a new and simple transmission scheme in which the sensor transmissions are ordered according to the magnitude of their measurements, and the sensors with small magnitude measurements, smaller than a threshold, do not transmit. A bound on the error of approximation is derived, which can be utilized to dynamically determine the threshold such that a trade-off between the accuracy of the approximation and the energy savings can be maintained. Through the numerical results, we show that our approach can be very energy efficient with only a negligible estimation error introduced.**

***Index Terms*—energy efficiency, ML-estimation, sensor network, ordered transmissions**

I. Introduction

A typical wireless sensor network [1], [2] usually consists of a large number of sensing devices that are capable of probing the environment and reporting the collected data, using a radio, to a fusion center (FC), as shown in Fig. 1. The advent of small-size, low-cost, low-power sensor technology has made it possible to deploy large sensor networks which can perform a comprehensive set of functions in a variety of application areas, such as environmental monitoring, military surveillance and intelligent transportation [3]–[5].

This work was supported in part by a Grant from the Commonwealth of Pennsylvania, Department of Community and Economic Development, in part by the National Science Foundation under Grant No. CCF-0829958, and in part by the U.S. Army Research Office under grant W911NF-08-1-0449.

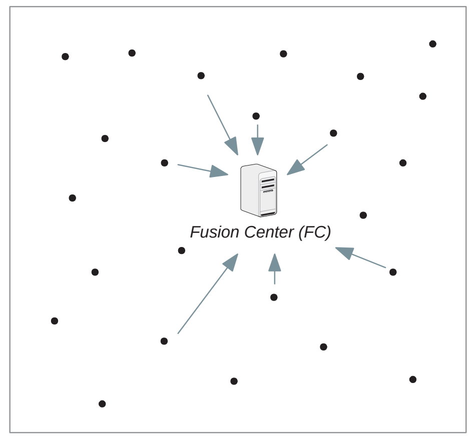

Fig. 1. Sensor network with fusion center.

Since sensors typically carry their own power in sensor network applications, it is crucial to reduce the energy consumption such that the lifetime of the sensor network can be maximized. Recently, several studies have considered energy efficient sensor networks [6]–[9]. However, these works either assume the sensors' capability to quantize and compress the data which may not be applicable for low cost large scale sensor network, or develop the scheduling policy of the sensors' activation which is very complicated, or have sensors exchange information randomly until reaching a consensus which results in a large number of transmissions. In this work, we focus on the approximation to the ML estimation of an unknown scalar parameter observed in Gaussian noise in the sensor network. Compared to the previous work, we achieve the energy savings by reducing the number of transmissions in a smart way. In particular, the FC may employ the approximation to the ML estimate which uses data from only some of the sensors, thus saving the transmission energy at the nodes that do not transmit. A new transmission scheme is described to achieve such an energy saving, in which sensors' transmissions are ordered [10], and can be halted when the difference between the ML estimate and the approximation is sufficiently small. Specifically, the ordered transmission is accomplished by delaying each sensor's transmission such that sensors with more important measurements will transmit earlier. This scheme is simple to implement without involving any cooperation between the sensors. Further, the FC can easily trade-off the energy saving and the error of the approximation.

This paper is organized as follows. The estimation problem under investigation is described in Section II. The ordered transmission approach is described in Section III. We also discuss the trade-off between the energy savings and the approximation. Simulation results are presented in Section IV, and conclusions are drawn in Section V.

II. Problem Setup

Consider a sensor network with a total of n sensors and a FC, which is set up to estimate an unknown (constant) scalar parameter $\theta \in \mathbf{R}$. The observation by each sensor is corrupted by additive noise, and the noisy measurement of the scalar θ made by sensor i is described by,

$$x_i = \theta + v_i. \tag{1}$$

We assume that $v_1, \cdots, v_n$ is a sequence of i.i.d. Gaussian random variables, each with zero mean and variance σ^2, i.e., $v_i \sim \mathcal{N}(0, \sigma^2)$. Consequently, we have that $x_i \sim \mathcal{N}(\theta, \sigma^2)$.

Each sensor sends its measurement to the FC directly. For simplicity, a perfect end-to-end transmission between any sensor and the FC is assumed, i.e., there is no data lost/distortion in the transmission. We assume that each sensor consumes the same amount of energy, e, to obtain the local measurement and to send its measurement to the FC. We also assume that sensors operate in orthogonal channels such that there is no inference.

It can be shown that the maximum-likelihood (ML) estimate of θ, given the measurements $x_1, \cdots, x_n$, is given by

$$\hat{\theta} = \frac{\sum_{i=1}^{n} x_i}{n}. \tag{2}$$

Note that $\hat{\theta}$ is unbiased, i.e., $E[\hat{\theta}] = \theta$, where $E[\cdot]$ denotes the expectation function. The variance of the above estimate is simply given by $\frac{\sigma^2}{n}$.

It can be seen from (2) that the ML estimate is obtained by averaging the measurements from all n sensors. For large n, having all sensors send their measurements to the FC may incur excessive energy consumption, which will shorten the life time of the whole network. Thus, it is of interest to develop a scheme that can reduce the number of sensor transmissions such that the energy can be saved.

III. Transmission Scheme

A. Basic Idea

For a given k ($1 \le k \le n$), if $|x_k| \approx 0$, we observe that

$$\sum_{i=1, i \neq k}^{n} x_i \approx \sum_{i=1}^{n} x_i, \tag{3}$$

where $|x_k|$ denotes the absolute value (or the magnitude) of x_k. Thus, from (2), we obtain

$$\hat{\theta} = \frac{\sum_{i=1}^{n} x_i}{n} \approx \frac{\sum_{i=1,,i \neq k}^{n} x_i}{n} \tag{4}$$

which indicates that when the magnitude of sensor k's measurement is close to 0, the FC can still obtain an accurate approximation of $\hat{\theta}$ as shown in (4), even if the sensor k does not send its measurement to the FC. Inspired by the above observation, we give the following transmission policy for the sensors:

Policy 1: For any given i ($1 \le i \le n$), the sensor i will transmit if and only if $|x_i| \ge \delta$, where $\delta \ge 0$ is a threshold determined by the FC.

To facilitate the analysis, we define the new estimate under Policy 1 as

$$\hat{\theta}_\delta = \frac{\sum\limits_{i=1, |x_i| \ge \delta}^{n} x_i}{n}. \tag{5}$$

As shown in the Appendix, the expectation and variance of $\hat{\theta}_\delta$ are given by

$$E[\hat{\theta}_\delta] = \theta - \int_{-\delta}^{\delta} x f_{(\theta, \sigma^2)}(x) dx, \tag{6}$$

and

$$Var[\hat{\theta}_\delta] = \frac{\sigma^2 + \theta^2 - E^2[\hat{\theta}_\delta] - \int_{-\delta}^{\delta} x^2 f_{(\theta,\sigma^2)}(x)dx}{n}, \tag{7}$$

where $f_{(\theta,\sigma^2)}(\cdot)$ denotes the probability density function (pdf) of a Gaussian random variable with mean θ and variance σ^2.

With (6) and (7), we can obtain the mean square error (MSE) as

$$\text{MSE}(\hat{\theta}_\delta) = Var[\hat{\theta}_\delta] + \left(\int_{-\delta}^{\delta} x f_{(\theta,\sigma^2)}(x)dx\right)^2. \tag{8}$$

We also have the following lemma.

Lemma 1: The energy saving produced under Policy 1 is non-decreasing with the threshold δ.

The correctness of Lemma 1 is straightforward, and the proof is omitted here. A larger δ can save more energy as more sensors do not transmit; while a smaller δ is more likely to result in a better approximation $\hat{\theta}_\delta$ (a smaller $|\hat{\theta} - \hat{\theta}_\delta|$). Thus, it is crucial to determine an appropriate δ to achieve a good trade-off between the energy saving and the error of the approximation of ML estimate. However, with θ to be estimated (unknown to sensors and the FC), the distribution of x_i is only partially known, and thereby it is very hard to determine value of δ directly.

B. Ordered Transmission

Now, we propose a transmission scheme in which the threshold δ can be dynamically determined for the desired trade-off between the amount of energy savings and the difference between $\hat{\theta}$ and $\hat{\theta}_\delta$. The main feature of this scheme is that the transmissions of all sensors are ordered according to the magnitudes of their measurements. Specifically, each sensor obtains its measurement of θ at the same time, but does not send its measurement to the FC immediately. Instead, it sends the measurement after a delay which is decreasing with the magnitude of its measurement. That is, the sensors having a larger magnitude measurement will transmit earlier.

Assume that $|x_1| \geq |x_2| \geq \cdots \geq |x_n|$, which indicates that the 1st sensor will transmit first, the 2nd sensor next, and so on. Define t_i as the delay for the i-th sensor. Generally, t_i can be written as,

$$t_i = g(|x_i|), \tag{9}$$

where $g(\cdot)$ is a monotonically decreasing function. A simple example function g is $t_i = \frac{1}{|x_i|}$. Note that this function g can be pre-designed for all sensors, prior to operation, and thereby the system does not need any further communication between the sensors and the FC to arrange the sensors' transmissions.

Now we explain how the FC can dynamically determine the threshold δ. For any given k ($1 \leq k \leq n$), upon receiving the k-th sensor's measurement, the FC updates the estimate of θ as

$$\hat{\theta}_k = \frac{\sum_{i=1}^{k} x_i}{n} \tag{10}$$

At this moment, there are $n-k$ sensor that have not transmitted their measurements to the FC. Specifically, they are the $(k+1)$-th sensor, the $(k+2)$-th sensor, $\cdots$, and the n-th sensor. According to the transmission scheme described above, $|x_k|$ is larger than the magnitudes of all these $n-k$ sensors' measurements, i.e., for any j ($k+1 \leq j \leq n$),

$$|x_k| \geq |x_j| \tag{11}$$

Recall that

$$\hat{\theta} = \frac{\sum_{i=1}^{k} x_i + \sum_{i=k+1}^{n} x_i}{n}. \tag{12}$$

Therefore, with (10), (11), and (12), we have the following inequality,

$$\hat{\theta}_k - \frac{(n-k)|x_k|}{n} \leq \hat{\theta} \leq \hat{\theta}_k + \frac{(n-k)|x_k|}{n}, \tag{13}$$

which can be re-written as

$$|\hat{\theta} - \hat{\theta}_k| \leq \frac{(n-k)|x_k|}{n}. \tag{14}$$

Equation (14) indicates that upon receiving sensor k's measurement, the FC is able to bound the difference between its current estimate ($\hat{\theta}_k$) and the unconstrained ML estimate $\hat{\theta}$ obtained from all sensor measurements.

If k is increased, $|x_k|$ decreases. When $\frac{(n-k)|x_k|}{n}$ is sufficiently small, $|\hat{\theta} - \hat{\theta}_k|$ will become sufficiently small from (14) , which indicates that the current estimate is good enough. Then, the FC can halt the transmissions of the remaining $n-k$ sensors, by setting $\delta = |x_k|$ and broadcasting this information to them. Consequently, these $n-k$ sensors will not transmit their measurements. The error of the approximation can be evaluated by substituting $\delta = |x_k|$ into (14),

$$|\hat{\theta} - \hat{\theta}_\delta| \leq \frac{(n-k)\delta}{n} \quad (15)$$

The resulted energy savings is given by

$$E_s = (n-k)e \quad (16)$$

where e is the amount of energy each sensor consumes for transmitting the measurement to the FC.

It can be seen from the above that the ordered transmission scheme is simple and easy to implement.

IV. Numerical Results

In this section, we present some numerical results to show how the trade-off between energy and the error of approximation can be achieved by implementing the proposed scheme. We simulate a network with $N = 200$ sensors. We let $\mu = 5$, $\sigma = 8$, and $e = 1$ unit of energy. We compute the normalized saving which is given by $\frac{E}{ne}$.

First, we show how the bias ($|\theta - E[\hat{\theta}_\delta]|$) and the MSE of $\hat{\theta}_\delta$ change versus the energy savings (δ varied from 0 to 5). The energy saving is represented by the percentage of sensor transmissions avoided under our proposed transmission scheme. As illustrated in Fig. 2, the proposed ordered-transmission scheme can bring a dramatic energy saving with a relatively small bias (($|\theta - E[\hat{\theta}_\delta]|/\theta$). The MSE of $\hat{\theta}_\delta$ versus the energy savings is shown in Fig. 3. Since the MSE of the ML estimate is given by $\sigma^2/n = 0.32$ in our setting, we observe that the proposed scheme can achieve almost the same MSE to the ML estimate for up to 25% energy savings, and a very close MSE for even larger energy savings.

Next, we illustrate how the FC can adjust the trade-off between the estimation performance and the energy saving. The curve of energy saving (per unit energy) versus the normalized error of the approximation ($|\hat{\theta} - \hat{\theta}_\delta|/\hat{\theta}$) is plotted in Fig. 4. It can be seen that the energy saving decreases as the error of the approximation decreases.

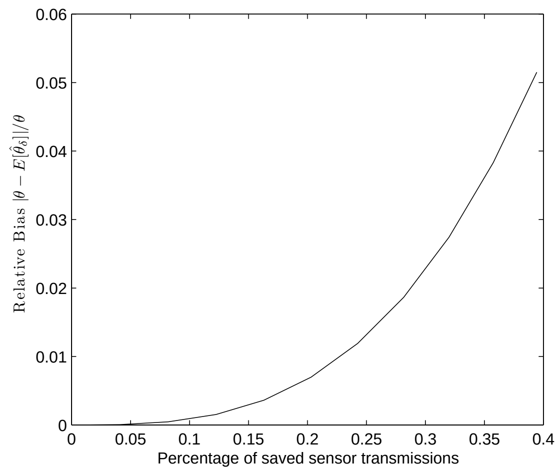

Fig. 2. Relative bias versus the energy savings.

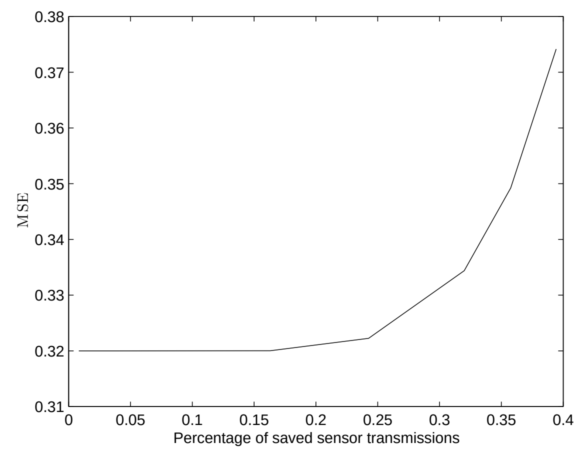

Fig. 3. MSE versus the energy savings.

V. Conclusion

In this paper, we consider estimation of an unknown scalar parameter in Gaussian noise in a sensor network, and discuss a new energy-efficient approach to obtain the approximation of the traditional ML estimate. In our approach, sensor transmissions are ordered according to the magnitude of measurements. Sensors with large magnitude measurements will transmit earlier, and those with small magnitude measurements, smaller than a threshold, will not transmit. Compared to the ML estimate, our approach saves energy by reducing the number of sensor transmissions. We also derive a bound on the approximation error which can be utilized to dynamically determine the threshold such that an

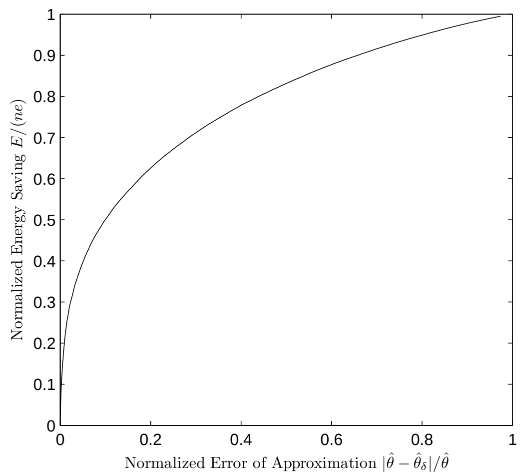

Fig. 4. Energy saving versus the error of approximation.

appropriate trade off between the energy savings the the accuracy of approximation can be maintained. Numerical results show that our approach can be very energy efficient with only a negligible error introduced.

VI. Appendix

In the Appendix, we show how we obtain (6) and (7).

Since $x \sim \mathcal{N}(\theta, \sigma^2)$, the pdf of x is given by

$$f_{(\theta,\sigma^2)}(x) \triangleq \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\theta)^2}{2\sigma^2}} \tag{17}$$

For any given i $(1 \le i \le n)$, we define

$$y_i = \begin{cases} x_i & \text{if } |x_i| \ge \delta; \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

Then, from (2), we have

$$\hat{\theta}_\delta = \frac{\sum_{i=1}^{n} y_i}{n} \tag{19}$$

Since the x_i's are i.i.d, the y_i's are also i.i.d. For simple notation, we drop the subscript i of x_i and y_i from now on. Then, we have

$$E[\hat{\theta}_\delta] = E[y], \tag{20}$$

and

$$Var[\hat{\theta}_\delta] = \frac{Var[y]}{n}. \tag{21}$$

It can be seen from the above equations that to obtain the expectation and the variance of $\hat{\theta}_\delta$, we need to know the pdf of y. To obtain the pdf of y, we first derive the cumulative density function (CDF) of y, denoted by $F_y(y)$. We have 3 cases as below.

If $|y| \ge \delta$, we have

$$\begin{aligned} F_y(y) &= \text{Prob}(X \le y) \\ &= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{y} e^{-\frac{(x-\theta)^2}{2\sigma^2}} dx. \end{aligned} \tag{22}$$

If $-\delta \le y < 0$, we have

$$F_y(y) = \text{Prob}(x \le -\delta) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{-\delta} e^{-\frac{(x-\theta)^2}{2\sigma^2}} dx. \tag{23}$$

If $0 \le y < \delta$, we have

$$F_y(y) = \text{Prob}(x \le \delta) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\delta} e^{-\frac{(x-\theta)^2}{2\sigma^2}} dx. \tag{24}$$

Denote the CDF of x by $F_x(x)$, we can combine (22),(23) and (24), and write the CDF of y as

$$F_y(y) = \begin{cases} F_x(y) & |y| \ge \delta; \\ F_x(-\delta) & -\delta \le y < 0; \\ F_x(\delta) & 0 \le y < \delta. \end{cases} \tag{25}$$

By taking the derivative of the CDF function of y given in (25) with respective to y, we obtain the pdf of y can be written as

$$f(y) = \begin{cases} f_{(\delta,\sigma^2)}(y) & |y| \ge \delta \\ (F_x(\delta) - F_x(-\delta))\, w(y) & y = 0 \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

where $w(\cdot)$ is the Dirac-delta function.

Thus the mean and the variance of y is given by

$$E[y] = \int_{-\infty}^{\infty} y f(y) dy, \tag{27}$$

and

$$Var[y] = \int_{-\infty}^{\infty} y^2 f(y) dy - E^2[y] \tag{28}$$

With (20), (21), (27) and (28), we can then easily obtain (6) and (7).

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