Near–Optimum Detection in Synchronous Code-Division Multiple-Access Systems

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Abstract—Communication networks employing code division multiple access include applications where several packets of information are transmitted synchronously and simultaneously over a common channel. In this paper, we consider the problem of simultaneously demodulating every packet from such a transmission. A nonlinear detection scheme based on a linear complexity multistage multiple-access interference rejection algorithm is studied. A class of linear detectors are considered as constituting the first stage for the multistage detector. A bit-error probability comparison of the linear and multistage detectors is undertaken. It is shown that the multistage detectors are capable of achieving considerable improvements over the linear detectors, particularly in near–far situations, i.e., in the demodulation of weak signals in the presence of strong interfering signals. This problem has been of primary concern for currently operational CDMA systems. Higher bandwidth utility factors are also achieved by the use of such detectors as will be demonstrated by some representative examples.

I. INTRODUCTION

In a code-division multiple-access (CDMA) system, several packets of information are transmitted simultaneously over a common channel using preassigned code waveforms. Each of these packets could be destined to a single receiver at a relay station (as in multihop radio networks) or different receivers (as in satellites transmitting to several earth stations). In either case, there are a number of applications where the receiver is equipped with a knowledge of the codes of all of the users. It is then required to demodulate the packets of information, upon reception of the sum of the simultaneously transmitted signals in the presence of additive noise.

Different approaches to the multistage demodulation problem have been considered so far. The conventional approach consists of demodulating each signal using the corresponding single-user detector (matched filter), thereby ignoring the multiple-access interference, or equivalently, ignoring the cross-correlations between the modulating signals of different users. While this approach has the virtue of being simple to implement, it has two major shortcomings. First, since low cross-correlations between a given number of signals can be achieved only at the expense of an increased bandwidth, it is not surprising that reliable performance from the conventional detector has been possible only for low bandwidth efficiencies [1], [3], [4]. Second, since error probability degrades exponentially as the interfering signal strengths increase, this detection scheme is highly vulnerable to near–far effects [7], [19], [21], [22].

On the other hand, the study of the optimum demodulator has shown that while significantly superior performance over the conventional detector is possible, it can be obtained only at a marked increase in computational complexity which is exponential in the number of users [21]. Since a CDMA system could potentially have a large number of users, this solution may prove, in a number of situations, to be impractical and too expensive to implement. Furthermore, most CDMA systems with a current need for only a small number of users may require an expansion capability without a steep increase in computational requirements.

The primary objective of this paper is to address the need for low-complexity detectors which perform reliably in high bandwidth efficiency situations and are robust to near–far effects. Attention is focused on the symbol-synchronous system for the following reasons. First, this system finds application in multicast networks which require point (central-station)-to-multipoint communication. Consider for instance, a satellite simultaneously transmitting a different message (or a set of messages) to each of several receiving ground stations. Another application would be in central station-to-central-station communications which is essential in multihop radio networks [16] such as cellular radio and some satellite networks. Second, although this system is a special case of the asynchronous system, the simplicity of the synchronous problem allows a thorough comparative error probability analysis of the suboptimum linear and multistage detectors in relation to the optimum detector. Finally, since the multistage detector can be generalized to the asynchronous situation in a natural way [19], and so can the optimum [21] and the linear detector [7], the performance of each of these detectors would be indicative of their asynchronous counterparts, with a judicious choice of signature waveforms.

The synchronous multistage demodulation has been previously considered by Timor in [14], [15] for noncoherent frequency-hopped FSK systems and by Horwood and Gaglardi in [5] and Schneider in [12], [13] for coherent CDMA systems. In particular, Schneider [12] proposed the decorrelating detector, the primary feature of which is its performance invariance to the signal energies of the interfering
users. More recently, Lupas and Verdu in [6], [8] recognized the exponential complexity of the optimum solution and proposed a linear decorrelating class of detectors together with the optimal linear detector, both of which require a real-time computational complexity which grows only linearly with the number of users. It was shown that a significant improvement over the conventional detection scheme could be achieved by the use of these detection schemes.

However, restricting the decision statistics to be linear functions of the sufficient statistics proves to be a rather stringent requirement and hinders the achievement of near-optimum performance in a number of situations, corresponding particularly to the demodulation of relatively weak signals. In this paper, we show that a multistage detector can improve upon the linear detectors in such situations. The decision statistics involved in this demodulation scheme are nonlinear functions of the sufficient statistics. However, the computational complexity is linear in the number of users, thereby retaining the computational advantage that the linear detectors have to offer.

The performance measure of interest in this paper is the bit-error probability. Exact expressions for the error probabilities are obtained for the optimum as well as the two-stage detector. The expression for the error probability of the optimum detector is presented here for the first time. In addition, a new approach is used to obtain bit-error probability of the two-stage detector which employs as the first stage, a linear decorrelating detector. The evaluation of error probability in this case is significantly simpler, thereby allowing a more thorough analysis than was possible for the two-stage detector with the conventional first stage for asynchronous systems in [19], for which the bit-error probability evaluation was numerically tractable for only one two-user channel.

In this paper, numerical examples of two-to-five user channels are considered. Evaluation of the error probability is carried out for the optimum detector, the two-stage detector as well as the linear detectors. In addition, since the error probability analysis for a higher stage detector is analytically tractable, some results from a Monte-Carlo simulation are presented to illustrate the performance of the three-stage and the four-stage detectors.

II. OPTIMUM DETECTOR

Let us assume that $K$ packets of information bits are transmitted simultaneously. The signature waveform employed to transmit the $k$th packet is denoted as $s_k(t)$, assumed time-limited to a bit duration $T$. This waveform is then used to antipodally modulate the $k$th packet of information bits. In a typical system, the transmitter at the central-station then transmits each of the $K$ modulated signals simultaneously in a symbol-synchronous fashion so that the signal at the receiver, denoted $r(t)$, can be written as

$$r(t) = \sum \sum_{k=1}^{K} b_k(t) s_k(t - iT) + n_t$$

(2.1)

where $b_k(t)$ denotes the $k$th user's bit in the $i$th time interval and $n_t$ represents the additive white Gaussian noise with a spectral density with height $\sigma^2$. The received energy of the $k$th signature waveform is denoted as $E_k$, thus allowing for unequal energies. This allows the transmitter to transmit a signal meant for a more distant receiver with a higher energy. Assuming that all possible information sequences are independent and equally likely, and defining $b^{(i)} = [b_1^{(i)}, b_2^{(i)}, \ldots, b_K^{(i)}]^T$, it is easy to see that an optimum decision on $b^{(i)}$ is a one-shot decision because it requires the observation of the received signal only in the $i$th time interval. Without loss of generality, we will therefore focus attention on $i = 0$ and drop the time superscript and consider the demodulation of the vector of bits $b = [b_1, b_2, \ldots, b_K]^T$ with the observation of the received signal in the time-interval $[0, T]$.

The optimum or the maximum likelihood decision on $b$ is chosen as $\hat{b}^* = [\hat{b}_1^*, \hat{b}_2^*, \ldots, \hat{b}_K^*]^T$ which maximizes the log-likelihood function, (see [17] or [10] for instance) can be expressed as

$$\hat{b}^* = \arg \left\{ \max_{b \in \{-1, +1\}^K} \left[ 2 \int_0^T r(t)s(t, b) \, dt - \int_0^T s^2(t, b) \, dt \right] \right\}$$

(2.2)

where $s(t, b) = \sum_{k=1}^K b_k s_k(t)$. The optimum decision can also be written as [8]

$$\hat{b}^* = \arg \left\{ \max_{b \in \{-1, +1\}^K} \left[ 2y^T b - b^T H H b \right] \right\}$$

(2.3)

where $y = [y_1, y_2, \ldots, y_K]^T$ is the vector of sufficient statistics with

$$y_k = \int_0^T r(t)s_k(t) \, dt, \quad \text{for } k = 1, 2, \ldots, K.$$

(2.4)

Further, the $(k, l)$th element of the matrix of cross-correlations $H$ is defined as

$$h_{kl} = \int_0^T s_k(t)s_l(t) \, dt.$$ 

(2.5)

For the rest of the paper, we will denote the $(k, l)$th element and the $k$th row of any matrix $M$ with the corresponding lowercase symbols $m_{kl}$ and $m_k$, respectively. The maximization problem in (2.3) was shown to be NP-hard [8]. That is, no algorithm which solves (2.3) in polynomial time in $K$ is known. While the exponential complexity of (2.3) may be acceptable in CDMA systems with a small number of users, it is prohibitive in large CDMA systems with the number of users in excess of 10 or 20, depending on the transmission rate, the available computing power, and the type of coherent modulation employed. For example, a 10-user, 100 kb/s system with QPSK signaling would require up to $10^5$ million computations of the likelihood function per second!
III. MULTISTAGE DETECTOR

In this section, we derive a low-complexity suboptimum approach to the maximization in (2.3). In particular, a multistage solution is considered [19]. Let the estimate of $b$ in the $m$th stage of the iteration be denoted as $\hat{b}(m) = [\hat{b}_1(m), \hat{b}_2(m), \ldots, \hat{b}_K(m)]^T$. Consider the $(m+1)$st stage estimate of the $k$th user's information bit $b_k$ as being

$$\hat{b}_k(m+1) = \arg \max_{z_k(m+1) \in \{\pm 1\}} \left\{ \frac{2y^T b - b^T \mathbf{H}b}{\sqrt{Q \lambda_k E_k}} \right\},$$

(3.6)

for $m \geq 1$. It is easily shown that

$$\hat{b}_k(m+1) = \text{sgn}[z_k(m)],$$

(3.7)

where $z_k(m)$ is the $m$th stage statistic for the $k$th user given as

$$z_k(m) = y_k - \sum_{j \neq k} j_k(m) h_{jk}.$$  

(3.8)

In demodulating the information bits of all the users, the maximization of (3.6) is performed for each $k = 1, 2, \ldots, K$. The $(m+1)$st stage estimate of $b$ can be written as the sign of the $(m+1)$st stage vector of decision statistics $z(m) = [z_1(m), z_2(m), \ldots, z_K(m)]^T$. Let

$$\hat{b}(m+1) = \text{sgn}[z(m)] = \text{sgn}[y - (\mathbf{H} - \mathbf{E})\hat{b}(m)],$$

(3.9)

where the diagonal matrix $\mathbf{E}$ has the energies of the modulating signals for its diagonal elements so that $\text{diag}(\mathbf{E}) = \{E_1, E_2, \ldots, E_K\}$. From the definition of the sufficient statistic in (2.4), it is easily shown that

$$y = \mathbf{H}b + \eta = \mathbf{E}b + I(b) + \eta,$$

(3.10)

where $\eta$ is a zero-mean Gaussian noise vector with a covariance matrix $\sigma^2 \mathbf{H}$ and $I(b) \triangleq (\mathbf{H} - \mathbf{E})b$ represents the multiple-access interference vector. Substituting (3.10) in (3.9), the expression for the $(m+1)$st stage estimate of $b$ is given as

$$\hat{b}(m+1) = \text{sgn}[z(m)] = \text{sgn}[\mathbf{E}b + I(b) - \hat{b}(m) + \eta].$$

(3.11)

The result in (3.11) has a simple interpretation. The $(m+1)$st stage estimate of $b$ is obtained as the sign of the $m$th stage statistics which in turn is obtained by subtracting from the sufficient statistic $y$, the estimate of the multiple-access interference based on the $m$th stage estimate of $b$.

IV. FIRST STAGE FOR MULTISTAGE DETECTOR

The development of the multistage solution described in the last section does not specify the first stage which delivers the initial estimate of the bits $\hat{b}(1)$. The choice of the first stage will prove to be important not only in the performance of the multistage detector, but also in simplifying the error probability analysis. We will focus attention on the class of linear detectors and in particular elaborate on the class of decorrelating detectors [8].

A. Linear Detectors

This section consists of a summary of the extensive results obtained for the conventional, decorrelating and the optimum linear detectors by Lupas and Verdu in [8]. This discussion is motivated by the fact that the properties of the first stage are essential to the study of the multistage detector. Moreover, we also undertake a comparative error probability analysis of these linear detectors as well as the optimum and the two-stage detectors in Section V. Noting from (2.3) that $y$ constitutes the vector of sufficient statistics, a linear detector denoted by a $K \times K$ matrix $\mathbf{L}$, is such that

$$\hat{b}(1) = \text{sgn}[\mathbf{Ly}] = \text{sgn}[\mathbf{z}(0)].$$

(4.12)

The conventional, decorrelating and the optimum linear detectors belong to this class.

Let us start by defining the two central performance measures that were used both to obtain the linear detectors and characterize their properties in [8]. The notion of asymptotic efficiency was introduced in the context of multiuser communication by Verdu in [22] to characterize the multiple-access limitation of multiuser detectors. If the $k$th user error probability is denoted as $P_k(\sigma)$, the corresponding asymptotic efficiency denoted as $\gamma_k$, is defined as

$$\gamma_k = \sup \left\{ 0 \leq r \leq 1; \lim_{\sigma \to 0} \frac{P_k(\sigma)}{Q^{\frac{1}{\sqrt{\lambda_k E_k}}}} < +\infty \right\}$$

(4.13)

so that the error probability goes to zero exponentially at the rate of the single-user error probability with energy $\lambda_k E_k$. The other performance measure, introduced in [8], referred to as near–far resistance, and denoted $\lambda_k$, denotes the asymptotic efficiency minimized over the energies of all interfering users. A nonzero near–far resistance denotes a nonzero asymptotic efficiency regardless of the interfering signal strengths. In this case, no matter how strong the multiuser interference, the bit-error probability goes to zero exponentially at a rate at least equal to that of the single-user error probability with energy $\lambda_k E_k$.

The conventional approach to the demodulation in multiuser channels relies entirely on a well-designed signal constellation by ignoring the multiple-access interference. Under such an assumption the optimum detector corresponds to the transformation $\mathbf{L} = \mathbf{I}$, the identity matrix. This detector has an asymptotic efficiency that is equal to zero in high-bandwidth efficiency and/or near–far situations and is therefore not near–far resistant [8].

When the modulating signals of all the users are linearly independent\footnote{No signal belongs to the subspace spanned by the other signals.} (note that this is a mild restriction), a decorrelating detector corresponding to $\mathbf{L} = \mathbf{H}^{-1}$, was first proposed by Schneider in [12]. This detector was proposed as a natural strategy to eliminate the multiple-access interference from the vector of sufficient statistics given in (3.10) so that

$$\mathbf{z}(0) = \mathbf{H}^{-1} y = b + \mathbf{H}^{-1} \eta.$$  

(4.14)

More recently, it was demonstrated in [8] that this detector corresponds to the maximum likelihood detector when the
energies are unknown to the receiver. It was also shown to be optimally near–far resistant, i.e., it achieves the highest worst case asymptotic efficiency over interfering signal energies. Since the decorrelator is obtained via a maximin approach, it is vulnerable to being conservative in environments where the signal energies remain fixed for a packet duration or vary slowly enough to lend themselves for easy tracking [11], [20].

A considerable portion of the work in [8] is devoted to obtaining the optimum linear detector where optimality indicates the maximization of the asymptotic efficiency. A procedure for finding this detector is detailed by Lupas and Verdu in their paper. Since the optimum linear detector solves the same problem as that of the multistage detector proposed in this paper, it is worthwhile to consider its properties. In particular, consider the necessary and sufficient conditions obtained in [8] on the signal energies and normalized cross-correlations under which the optimum linear detector achieves optimum asymptotic efficiency for the kth user, given as

$$\sqrt{E_k} > \max_{j=1,\ldots,K} \left( \frac{1}{|r_{kj}|} \sum_{i \neq k} \sqrt{E_i} |r_{ij}| \right)$$

(4.15)

where \( R \) represents the matrix of normalized cross-correlations, i.e., \( H = E^{1/2} R E^{1/2} \). Note that if the kth user satisfies (4.15), then \( \sqrt{E_k} > \sqrt{E_j} / |r_{kj}| > \sqrt{E_j} \), \( j \neq k \). Therefore no more than one out of K users achieves optimum asymptotic efficiency for a given set of signal energies and cross-correlations, i.e., for a given operating point. Also observe from (4.15) that the desired user has to be sufficiently strong to be demodulated with optimum asymptotic efficiency. Attempting further improvement for this strong user may therefore be superfluous. On the other hand, the other K – 1 users will not satisfy (4.15). It is the demodulation of these signals, possibly relatively weak, that will be of interest in this work. Indeed, the near–far problem pertains to this very situation, i.e. the demodulation of relatively weak signals in the presence of strong interferers. As was noted in [8] there is little or no improvement in going from the decorrelating to the linear optimum detector in such cases.

B. Decorrelating-Type Detectors

Motivated by the desirable properties of the decorrelator [8], we define in this section, a class of decorrelating-type detectors. The justification for this class of detectors is two-fold: these detectors retain the robust properties of the decorrelator and the error probability analysis is computationally much simpler for the two-stage detector which employs a member of the decorrelating-type class as the first stage than it is for any other first stage.

Let us denote the class of decorrelating-type detectors as \( C(H) \). A linear detector \( V \) is said to belong to \( C(H) \) if \( V \) is of the form

$$V = \begin{bmatrix} h_{11}^{H} \\ h_{21}^{H} \\ \vdots \\ h_{K1}^{H} \end{bmatrix}$$

(4.16)

where \( h_{ij}^{H} \) is the ith row of some generalized inverse of \( H \) belonging to the class of generalized inverses [2] denoted as \( I(H) \). In particular, if the ith signal is linearly independent, then \( h_{ij}^{H} \) can be chosen to be any generalized inverse if and say the jth user is linearly dependent, then \( h_{ij}^{H} \) is restricted to be a specific member of \( I(H) \) corresponding to the unique Moore-Penrose generalized inverse.\(^3\)

Two justifications for the use of the class \( C(H) \) of initial choices will now be given. First, a generalization of the decorrelating detector \( H^{-1} \) was proposed in [8] in the situation where the linear independence assumption does not hold. This general class was chosen to be the set of generalized inverses \( I(H) \) of \( H \). Performance analysis of this class of decorrelating detectors has revealed that the asymptotic efficiency of a linearly independent user was independent of the particular decorrelating detector and also independent of the signal energies of the interfering users. By using a member \( V \) of the class of detectors \( C(H) \) in demodulating the ith user’s bit, the decision statistic of this linear detector is only a function of the ith row of \( V \). Although the class of linear detectors \( V \in C(H) \) does not correspond to the class of generalized inverses, it is indeed a meaningful choice since each row of \( V \) is the corresponding row of some generalized inverse. Notice also that the Moore–Penrose generalized inverse, denoted as \( H^+ \), also belongs to \( C(H) \). Further, if all the users are linearly independent, the only element of \( C(H) \) is \( H^{-1} \) which perhaps is of primary interest in most well-designed CDMA systems. For the second justification the reader is referred to the lemma in the Appendix. This lemma will play a key role in simplifying the derivation as well as computation of the probability of error of the two-stage detector with a decorrelating-type first stage.

V. PROBABILITY OF ERROR ANALYSIS

The use of CDMA is motivated in part due to the fact that a multiple-access system has to operate not only in the presence of additive thermal noise but also in the presence of other external disturbances such as adjacent channel interference, multipath, jamming, etc. For a CDMA system with well-designed codes, the decoding operation (for instance, "despreading" in spread-spectrum systems), has the effect of spreading the spectrum of these disturbances which will then effectively raise the noise level. Therefore, even if the thermal noise level is low, it is of interest to evaluate such systems by their bit error probabilities for low as well as high values of SNR, depending on the levels of the external disturbances. In this sense, the probability of bit-error is a more detailed performance measure than asymptotic efficiency as defined in (4.13). In the sections which follow, the bit-error probabilities of the optimum, linear and the two-stage detectors are derived.

A. Optimum Detector

It was noted in Section II that the optimum detector has a prohibitive computational complexity. However, in order

\(^2\) A is a generalized inverse of \( B \) if \( ABA = A \) and \( BAB = B \).

\(^3\) A is a Moore–Penrose generalized inverse of \( B \) if it is a generalized inverse of \( B \) and satisfies \( AB = B \) and \( BA = A \) being Hermitian.
to show that the linear complexity suboptimum detectors are near-optimum, we must analyze the performance of the optimum detector. In this section, the bit-error probability of the optimum detector given by (2.3) is derived. Without loss of generality, consider the error probability of the first user. Denote the transmitted vector of bits as \( b^* = [b_1^*, \ldots, b_K^*]^T \) and \( y \), the vector of sufficient statistics as \( y(b^*) \) to express its dependence on \( b^* \). Let the set of all decisions which are erroneous in the bit of the first user be denoted as \( S(b^*) \). Conditioning on \( b^* \) and denoting the error probability of the first user as \( P_1^* \), we have

\[
P_1^* = E_{b^*} \left\{ \Pr \left[ b^* \in S(b^*) \right] \right\}
\]

(5.17)

with \( b^* \) being the solution of (2.3) and where \( E_{b^*} \) denotes the expectation over the ensemble of identical, uniformly distributed \( b^* \in \{-1, +1\}^K \). Since each \( b' \) that belongs to \( S(b^*) \) represents a disjoint region in \( y(b^*) \)-space, the probability that \( b^* \) is a member of \( S(b^*) \) is equal to the sum of the \( 2^{K-1} \) probabilities that each member of \( S(b^*) \) is equal to the optimum decision. Therefore,

\[
P_1^* = E_{b^*} \left\{ \sum_{b' \in S(b^*)} \Pr \left[ b' \in S(b^*) \right] \right\}
\]

(5.18)

where we adopt the convention that the symbol \( \geq \) between vectors denotes "greater than equal to" component-wise. It can be verified that the \( j \)th element of the \( 2^K - 1 \) column vector \( c(b^*) \) is written as

\[
c_j(b^*) = C(b^*, D^{(j)}b^*)
\]

(5.21)

where the \( K \times K \) diagonal matrices \( D^{(j)} \) for \( i = 1, 2, \ldots, 2^K - 1 \) are defined such that the \( j \)th diagonal element of \( D^{(j)} \) is given as \((-1)^{a_{ij}}\), \( a_{ij} \) being the \((i,j)\)th element of the matrix \( A \). Further, the scalar \( C(b^*, x) \) for \( x \in \{-1, +1\}^K \) is defined as

\[
C(b^*, x) = \frac{1}{4} \left( b^{T} H b^* - x^{T} H x \right) - \frac{1}{2} (b^* - x)^{T} H b^*.
\]

(5.22)

Now note that for given \( b^* \) and \( b^* \), the probability \( \Pr \left[ A b^* \geq c(\cdot) \right] \) is equivalent to the computation of the integral of the multivariate normal density function of \( \nu \) over the polytope \( A b^* \geq c(\cdot) \). It is not difficult to show that this probability can be expressed in terms of sums and differences of multivariate normal distribution functions of dimension less than or equal to \( K \). This fact is illustrated by an example when \( K = 2 \). In this case, the matrix \( A \) will be

\[
\begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]

and the probability in the curly bracket in (5.20) can be written as shown at the bottom of the page. Clearly, each probability term in the above equation can be written as a bivariate or a univariate normal distribution function.

The optimum bit-error probability is used as a benchmark to compare the linear and the multistage detectors in Section V. Any claim to near-optimality of a suboptimum detector will have to be based on how closely the bit-error probabilities of these suboptimum schemes track optimum performance.

**B. Linear Detector**

In this section, we consider the bit error probability of the linear detector \( L \). Again, without loss of generality, consider the error probability of the first user. Denoting it as \( P_1^{(1)}(L) \), it is clear that for \( F \triangleq LH \),

\[
P_1^{(1)}(L) = P \left[ b_1(1) = 1 \mid b_1 = -1 \right]
\]

\[
= P[(Fb + \gamma)_1 > 0 \mid b_1 = -1]
\]

\[
\Pr[A \nu \geq c(\cdot)] = \begin{cases} 
\Pr[v_1 \geq c_2, v_2 \geq c_3] & \text{if } c_1 + c_2 \geq c_3 \\
\Pr[v_1 \geq c_2, v_1 + v_2 \geq c_3] + \Pr[v_2 \geq c_1, v_1 + v_2 \geq c_3] - \Pr[v_1 + v_2 \geq c_3] & \text{if } c_1 + c_2 < c_3.
\end{cases}
\]
where the second equality follows from (3.10) and (4.12) and \( \gamma \) is a zero-mean Gaussian random vector defined as \( \gamma \triangleq L \eta \) with covariance \( \sigma^2 G \) where \( G \triangleq L H L^T \). Let \( \beta = [b_2, b_3, \ldots, b_K] \) be the \((K - 1)\)-dimensional column vector that represents the bits that interfere with \( b_1 \). Denoting

the expectation over the ensemble of independent, uniformly distributed \( \beta \in \{-1, +1\}^{K-1} \) as \( E_{\beta} \), we have

\[
P^1(1)(L) = E_{\beta} \left[ P \left( \gamma_1 > f_{11} - \sum_{j=2}^{K-1} f_{1j} b_j \right) \right]
= 2^{1-K} \sum_{\beta \in \{-1, +1\}^{K-1}} \Phi \left( \frac{f_{11} - \sum_{j=2}^{K-1} f_{1j} b_j}{\sigma^2 g_{11}} \right)
\tag{5.23}
\]

where the last equality follows from the fact that \( \gamma_1 \) is a zero-mean Gaussian random variable with variance \( \sigma^2 g_{11} \). This expression will be used to compute the error probabilities for the conventional, decorrelating and linear detectors in the section on numerical results.

C. Two-Stage Detector with Decorrelating-Type First Stage

This section consists of the derivation of the error probability of the two-stage detector which employs a decorrelator-type linear detector \( V \), as the first stage. For convenience, we will continue to use \( F, G, \) and \( \gamma \) defined in the previous section but with the arbitrary linear transformation \( L \) replaced by the decorrelating-type transformation \( V \). Without loss of generality consider again the demodulation of \( b_1 \). The two-stage detector decision from (3.11) can be written as

\[
b_1(1) = \text{sgn}[z_1(1)]
\]

where the decision statistic \( z_1(1) \) is also obtained from (3.11) as

\[
z_1(1) = E_1 b_1 + \sum_{k=2}^{K} h_{1k} [b_k - \hat{b}_k(1)] + \eta_1. \tag{5.24}
\]

The first, second, and third terms will be referred to as the desired component, residual interference and additive noise, respectively. Note that the residual interference depends on \( \delta = \beta - \hat{\beta}(1) \) where \( \beta \) is defined in the previous section and \( \hat{\beta}(1) \) denotes the first stage estimate of \( \beta \) obtained by the linear detector \( V \). Therefore, we denote the residual interference as \( I(\beta, \delta) \) to explicitly show its dependence on \( \delta \) and hence on \( \beta \). The decision statistic can be written as

\[
z_1(1) = E_1 b + \eta_1 + I(\beta, \delta). \tag{5.25}
\]

The general strategy will now be outlined. The error probability is expressed as the expectation of the conditional error probability, conditioned on the bits \( b = [b_1, \beta^T]^T \) and on the error vector \( \delta \), to obtain

\[
\text{Pr}(\text{error}) = E_{b, \delta}[E_{\delta}(\text{Pr}[\text{error} | b_1, \beta, \delta])]. \tag{5.26}
\]

Next, using (5.25) in the probability expression on the right-hand side of (5.26), we have

\[
\text{Pr}[\text{error} | b_1, \beta, \delta] = \begin{cases} 
\text{Pr}[\eta_1 > E_1 - I(\beta, \delta) | b_1 = -1, \beta, \delta] & \text{if } b_1 = -1 \\
\text{Pr}[\eta_1 < -E_1 - I(\beta, \delta) | b_1, \beta, \delta] & \text{if } b_1 = +1.
\end{cases}
\tag{5.27}
\]

The rest of the derivation deals with the evaluation of the probabilities in (5.27). Then the expectation over \( \delta \) in (5.26) is evaluated by specifying the probability density function of the error vector \( \delta \). Finally, removing the conditioning on the bits is simply a matter of averaging simple random variables.

In order to compute the probability in (5.27), we require the probability density function of \( \eta_1 \) conditioned on \( \beta \). Since \( \delta = \beta - \hat{\beta}(1) \) and \( \hat{\beta}(1) = [b_2(1), \cdots, b_K(1)] \) with each first stage estimate given by the linear detector \( V \), we have from (3.10),

\[
\hat{b}_k(1) = \text{sgn}[v_k \mathbf{h}] = \text{sgn}[f_{1k} b + \gamma_k]. \tag{5.28}
\]

Therefore, given \( b \), the error vector \( \delta \) is entirely determined by \( \xi \triangleq [\gamma_2, \cdots, \gamma_K]^T \). Therefore, we need to determine the density function of \( \eta_1 \) conditioned on \( \xi \). Consider the \( K \)-dimensional zero-mean Gaussian random vector \( [\xi, \eta_1]^T \) which has a covariance matrix given as

\[
E \left[ \begin{array}{c} \xi \\ \eta_1 \end{array} \right] \left[ \begin{array}{c} \xi^T \\ \eta_1 \end{array} \right] = \sigma^2 \begin{bmatrix} G & f_{c1} \\ f_{c1}^T & h_{11} \end{bmatrix}
\]

where each of the blocks is derived from the decomposition of \( F \) and \( G \) given by

\[
F = \begin{bmatrix} f_{11} & f_{c1} \\ f_{c1} & F \end{bmatrix}, \quad G = \begin{bmatrix} g_{11} & \tilde{g}_{c1} \\ \tilde{g}_{c1} & G \end{bmatrix}
\]

where \( f_{c1} \) and \( \tilde{g}_{c1} \) are \((K - 1) \times 1\) column vectors and \( f_{c1} \) and \( \tilde{g}_{c1} \) are \(1 \times (K - 1)\) row vectors and \( F \) and \( G \) are \((K - 1) \times (K - 1)\) matrices. In particular, the cross-correlation between \( \xi \) and \( \eta_1 \) is given as \( E[\xi \eta_1] = f_{c1} \). However, since \( V \in C(H) \), and \( f_{c1} \) consists of the elements \( (V H)_{2,1}, \cdots, (V H)_{K,1} \), we have \( f_{c1} = 0 \), as a direct consequence of the lemma in appendix implying that, \( \xi \) is independent of \( \eta_1 \). Therefore for a given \( b \), since \( \delta \) is a function of \( \xi \) only, the additive noise \( \eta_1 \) is independent of \( \delta \) and hence of the residual interference. As a result, the probability density function of \( \eta_1 \) conditioned on \( \delta \) is the same as its marginal zero-mean normal density with variance \( \sigma^2 h_{11} = \sigma^2 E_1 \). Therefore,

\[
\text{Pr}[\eta_1 > E_1 - I(\beta, \delta) | b_1 = -1, \beta, \delta] = Q \left( \sqrt{\frac{1}{\sigma^2 E_1}} [E_1 - I(\beta, \delta)] \right) \tag{5.29}
\]

and

\[
\text{Pr}[\eta_1 < -E_1 - I(\beta, \delta) | b_1, \beta, \delta] = Q \left( \sqrt{\frac{1}{\sigma^2 E_1}} [E_1 + I(\beta, \delta)] \right) \tag{5.30}
\]
where $Q(\cdot)$ is the complementary error function so that
\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2} dt. \]

We now evaluate the expectation over $\delta$, in (5.26). Note that $\delta$ takes on $2^{K-1}$ possible values corresponding to $[\delta]_i \in [2(\beta), 0]$ depending on whether $[\tilde{\beta}(1)]_i$ is in error or not. Denote the set of all possible values of $\delta$ as $S(\beta)$ to show its dependence on $\beta$. For a given $\beta$, each realization of $\delta \in S(\beta)$ corresponds to $\tilde{\beta}(1)$ being equal to $\beta - \delta$. Therefore, the probability of such an event is equal to the joint density function of the first stage decisions $\tilde{\beta}(1)$ evaluated at $\beta - \delta$. From (5.28), it is easily seen that this joint density is equivalent to the probability that $[v_{2Y}, \cdots, v_{KY}]$ belongs to a certain hyperquadrant which depends on the specific value of $\beta - \delta$. It is clear from (5.28) that this probability can be expressed equivalently as the $(K-1)$-dimensional normal distribution function of $\xi$. Let us denote it as
\[ P_{b_\delta}(\beta, \delta) \triangleq \Pr[\tilde{\beta}(1) = \beta - \delta | b, \delta]. \quad (5.31) \]

Finally, we substitute (5.29) and (5.30) in (5.26), and write the expectation over $\delta$ explicitly using (5.31). Denoting the error probability as $P_1^{(2)}(V)$, we have from (5.26),
\[ P_1^{(2)}(V) = \frac{1}{2} E_{\beta} \left[ \sum_{\delta \in S(\beta)} P_{b_\delta}(\beta, \delta) \times \right. \]
\[ \left. \frac{1}{\sqrt{2\pi} E_1} \left( (E_1 - I(\beta, \delta)) + \sum_{\delta \in S(\beta)} P_{b_\delta}(\beta, \delta) \times \right. \right. \]
\[ \left. \left. \frac{1}{\sqrt{2\pi} E_1} (E_1 - I(\beta, \delta)) \right) \right]. \quad (5.32) \]

Clearly, the evaluation of this expression in (5.32) does not involve any numerical integrations and is therefore significantly more expedient than its counterpart in [19]. The evaluation of a $(K-1)$-dimensional Gaussian distribution function is, however, still required. For details on the efficient computation of the multivariate normal distribution function see [9].

Recall that the performance of each detector belonging to the class of decorrelating detectors is identical for independent signals, the performance measure being bit error probability. However, different choices of $V$ would yield different two-stage detector error probabilities, due to the fact that the performance of the two-stage detector is not only affected by the individual bit error probabilities of different users [which would be the same for all $V \in C(H)$] but also the joint distribution of the initial decisions.

D. Two-Stage Detector with Arbitrary Linear First Stage

A direct extension of the approach in Section V-C to the case where the first stage employed is any linear transformation, fails to yield a satisfactory method for the following reason. Note from (5.28) that, for a given $b, \xi$ completely determines $\delta$. Therefore, conditioning on $\delta$ in (5.26) can be replaced by conditioning on $\xi$. For the computation of the probabilities in that equation, we therefore require the probability density function of $\eta_1$ conditioned on $\xi$. In general, the random variables forming $\xi$ are correlated with $\eta_1$, since they are obtained by an arbitrary linear transformation of $\eta_1$, each component of which in turn is the result of integrating the product of the corresponding signal and a common white noise process over identical time intervals. Therefore, the probabilities in (5.26) would depend on $\xi$. Removing the conditioning on $\xi$ would therefore involve numerical computation of a multidimensional integral over the space $R^{K-1}$.

The approach in [19] for asynchronous systems with the conventional first stage can be generalized to the case of any linear first stage [18]. The generalization involves no conceptual difficulties and therefore will not be included here. Recall that this expression for the error probability requires only computation of one-dimensional numerical integrals over bounded intervals, thereby making it numerically tractable. For synchronous systems, only $(K-1)$-dimensional normal distribution functions need to be evaluated, as opposed to $2(K-1)$-dimensional distributions for asynchronous systems.

VI. NUMERICAL RESULTS

This section is a presentation of numerical results obtained from the computation of the bit-error probabilities of the optimum detector and the low-complexity suboptimum detectors. The primary objective is a comparative study to determine the best suboptimum detector as a function of the operating point, i.e., the normalized cross-correlations and relative energies of the modulating signals. Among the linear detectors, the conventional, decorrelating and the optimum linear demodulators are considered. The conventional detector is included to account for its "conventional" status and illustrate just how poorly it can perform; the decorrelator is a natural candidate for such a study, being optimum in the sense of near-far resistance; the usefulness of the optimum linear detector should be determined by the performance gain it achieves over the decorrelator. The two-stage detector which employs two different first stage detectors, decorrelating and conventional, is considered for the following reasons. The decorrelator is an excellent choice for the first stage due to its performance invariance to interfering signal energies. However, its implementation in asynchronous systems may involve large storage and long decoding delays—the decorrelating detector in this case being a noncausal $K$-input—$K$-output linear decorrelating filter [7]—that are unacceptable in some applications. On the other hand, while the conventional first stage is clearly inferior to the decorrelator, its use in the multistage detector in asynchronous systems results in an easily implementable detector with a small storage requirement and a short decoding delay [19]. Finally, the optimum error probability is used as a benchmark performance to decide the near-optimality of the suboptimum detectors. Further, it helps identify the need for more stages for those operating points where the performance of the two-stage detector falls short of being near-optimum.

To this end, the performances of the three-stage and four-stage detectors are obtained from a Monte-Carlo simulation study, details of which can be found in [18].

The rest of the section is organized into two subsections. The first deals with the two-user case which allows an exhaustive study of all detectors considered in this work. The
second subsection deals with a five-user system where the signature waveforms are chosen such that, the error probability evaluations will provide in addition to a comparison of the suboptimum schemes in the multiuser situation, results which will give an indication as to what one might expect in the corresponding asynchronous system.

A. Two-User Case

The two-user example is simple and yet illustrative of the salient features of the different detectors. Let us first begin with Fig. 1. The signature waveforms of the two users depicted in this figure have been previously considered [19], [21], [22] in the literature and correspond to the cross-correlation parameter $r_{12} = 1/3$. The signal strength of the desired user, which in our case is the first user, is fixed at 8 dB. The error probability of the desired user for each of the detectors is depicted as a function of $E_2/E_1$ ranging from −10 to 10 dB corresponding to the interfering signal energy being a tenth of the desired signal energy to ten times that level. Note that the behavior of the optimum detector error probability indicates that the near–far problem is not inherent of CDMA systems. The performance of user 1 actually improves as the interfering user becomes stronger. This is consistent with the finding of Verdu in [21] based on asymptotic efficiency analysis, that when the interfering users are sufficiently strong, it is the noise, rather than the randomness of the information of these users that is a primary source of errors. On the other hand, the error probability of the conventional detector, even in this low cross-correlation situation, suffers a severe degradation in relation to the optimum error probability as the interfering energy increases. This exhibits the severity of the near–far problem associated with conventional detectors in CDMA communications. Finally, the suboptimum detectors are near–optimum and there is little to choose between them in this case.

Consider now a situation that is more representative of a high bandwidth efficiency situation which would correspond to a higher cross-correlation, $r_{12} = 0.7$. Fig. 2 depicts the error probabilities in much the same set up as does Fig. 1, except for the difference in the value of $r_{12}$. It is clear from this figure that the suboptimum detectors no longer perform similarly, nor do they achieve near–optimum performance for all operating points. Notice that performance of the conventional detector is far worse than it was in Fig. 1. The explanation of the behavior of the conventional detector in Figs. 1 and 2 is straightforward. Since it ignores the multiple-access interference, it has an acceptable performance only for very low values of $E_2/E_1$ and degrades exponentially as this ratio increases. On the other hand, the error probability of the decorrelating detector remains invariant with interfering signal strength. The point to be noted here, however, is the considerable degradation of the decorrelator in relation to the optimum detector with increasing value of $r_{12}$ from Figs. 1 and 2. Meanwhile, it is seen that the optimum linear detector has a dual behavior. If the interfering signal strength is small enough, i.e., $E_2/E_1 < |r_{12}|$, a near–optimum performance is observed. Note that this is consistent with the condition in (4.15) that, for the two-user channel, the optimum linear detector achieves optimum asymptotic efficiency in this region. If the interfering signal in strong enough, i.e., $E_2/E_1 > |r_{12}|$, the optimum linear detector coincides with the decorrelating detector, and so do their error probabilities [8].

Let us now consider the multistage detector when the interfering signal is weaker than the desired signal. Observe that the two-stage detector does not necessarily perform better than the corresponding first stage for a relatively weak interfering signal. In this case, the interfering signal is not estimated well and hence is not rejected often enough for the effect of the successfully rejected interference to prevail over the effect of interference doubling corresponding to unsuccessful estimation of the interferer. On the other hand, the three-stage detector is clearly near–optimum. The reason for this is that since the weak interferer is estimated well by the two-stage, the effect of its successful rejection in the third stage dominates.

However, as the interfering signal strength increases, the two-stage detector performance improves dramatically and
approaches optimum performance when the interfering signal is stronger. It may be noted that improvements over the decorrelating (or the linear optimum) detector are more marked for larger correlation, i.e., comparing Fig. 1 to Fig. 2 where the decorrelator suffers from the conservativeness of a maximin solution. Clearly, in the high interfering signal energy situation, the interference rejection effect due to the successful estimation of the interference prevails over the interference doubling effect. The three-stage detector does not improve on the two-stage since the two-stage estimate of the weak interferer is not necessarily an improvement over the first stage. We conclude from this discussion that it helps to have a good first stage and also that between the two-stage and three-stage detectors, near--optimum performance can be obtained for any interfering signal strength.

Let us consider the situation in Fig. 3 where \( r_{12} \) is the same as in Fig. 2 but the desired signal-to-noise ratio is now 12 dB. The purpose of this example is to reconcile the results obtained here to the asymptotic results obtained in [8]. The relative performances of all the detectors remain essentially the same as in Fig. 2. Note that the decorrelator nearly achieves the optimum error probability for the worst case \( E_2/E_1 \), a phenomenon which was not apparent in Fig. 2, which is in accordance with the fact that the decorrelator has optimum near--far resistance. More importantly, notice the large difference between the optimum error probability for the worst case \( E_2/E_1 \) and its value for other energy ratios, such as high values corresponding to the near--far situation. In fact, it is for these operating points, that the two-stage detector achieves significant improvement over the decorrelator.

A final comment regarding the optimum linear detector. It can be seen from Figs. 1 and 2 that, even though the optimum linear detector performs best in the low \( E_2/E_1 \) region, its performance can be emulated somewhat closely by the conventional detector, at least in the two-user system. It is therefore of interest to consider the equivalent question for the general \( K \)-user system. Suppose that the \( k \)-th user

linear optimum detector asymptotic efficiency is equal to the optimum asymptotic efficiency. Recall from (4.15) that no other user can have optimum asymptotic efficiency. Using (4.15) we have

\[
\frac{1}{\sqrt{E_k}} \sum_{i \neq k} \sqrt{E_i} |r_{ik}| \leq \frac{1}{\sqrt{E_k}} \max_{j=1,\ldots,K} \left( \frac{1}{|r_{kj}|} \sum_{i \neq k} \sqrt{E_i} |r_{ij}| \right) < 1.
\]

Now the asymptotic efficiency of the \( k \)-th user for the conventional detector is given as

\[
\gamma_k = \max \left\{ 0, 1 - \frac{1}{\sqrt{E_k}} \sum_{i \neq k} \sqrt{E_i} |r_{ik}| \right\}.
\]

It is clear from (6.33) and (6.34) that if the operating point is such that the \( k \)-th user achieves optimum asymptotic efficiency using the optimum linear detector, then the \( k \)-th user achieves a nonzero asymptotic efficiency using the conventional detector. An exact quantification of this statement for the two-user example can be obtained using (6.34), so that

\[
\min_{E_2/E_1 \leq |r_{12}|} \gamma_1 = \left( 1 - r_{12}^2 \right)^2.
\]

Noting further that the optimum asymptotic efficiency for the first user (denoted \( \gamma_1^* \)) in a two-user system is equal to

\[
\min\{1, 1 + E_2/E_1 - 2|r_{12}|/\sqrt{E_2/E_1}\},
\]

it is easily seen that

\[
\max_{|r_{12}|} \frac{\max_{|r_{12}|} (\gamma_1^* - \gamma_1)}{|r_{12}|} = \frac{\max_{|r_{12}|} (1 - 2|r_{12}|)}{|r_{12}|} = \frac{1}{4}.
\]

In other words the maximum difference between linear optimum asymptotic efficiency and the asymptotic efficiency of the conventional detector for operating points which satisfy (4.15) for the two-user channel, is equal to 0.25. Since this maximum difference corresponds to \( |r_{12}| \) being equal to \( 1/\sqrt{2} \), the situation in Fig. 2 which corresponds to \( r_{12} = 0.7 \), depicts approximately the largest difference in performance between the conventional and the linear optimum detector. Since this difference is not very significant, we will exclude the optimum linear detector from the next subsection, with the understanding that it yields a marginally better performance than the decorrelator.

B. A Five-User Spread-Spectrum Example

This example is aimed at addressing a number of important issues that were discussed throughout this work. We will consider the modulating signals of five different users as being base-band signals derived from Gold sequences of length seven as shown in Fig. 4. A justification for the choice of this signal set is now in order. As was noted earlier, the linear and the multistage suboptimus approaches lend themselves to generalizations for asynchronous CDMA systems. Therefore, a comparative performance analysis of the suboptimum detectors for synchronous channels would
performance shows a clear improvement over the first stage, which is particularly marked as the interfering signals' strength increase, a phenomenon that was also observed in the two-user channel examples in Section IV-A. Fig. 5(d) depicts the simulated performance of the three-stage and the four-stage detector as well. This figure illustrates that while in general, improvements over the decorrelator are possible, the choice of the number of stages employed could be an important one since the two-, three-, and four-stage detectors perform better than the decorrelator to different degrees depending on the operating point.

VII. CONCLUSION

Optimum demodulation in CDMA systems is inherently a difficult problem requiring a computational complexity per demodulated bit that is exponential in the number of users. Single-user (conventional) demodulation suffers severe degradation due to the presence of multiple-access interference when the bandwidth has to be efficiently utilized and/or when the interfering signals are strong. Linear detectors have the virtue of linear computational complexity and are therefore simple to implement. Among these, the optimum linear detector and the decorrelating detectors alleviate the shortcomings of the conventional detector since their performance remains invariant to interfering signals' strengths. This invariant error probability is however a function of the signal cross-correlations and can be quite different from the optimum error probability, particularly for relatively weak users. The multistage detector proposed in this paper, based on a decorrelating first stage, was shown to perform significantly better than the decorrelating and the optimum linear detectors, in a number of situations of practical interest such as in high bandwidth utility and in near-far situations. In addition, although the multistage detector decision statistic is a nonlinear function of the sufficient statistics, it has a linear real-time computational complexity per demodulated bit.

APPENDIX

In this Appendix, we show that the additive noise and residual interference in (5.24) are statistically independent when the first stage decisions are made by a decorrelating-type detector defined in Section IV-B. It is this fact that is responsible for rendering the analysis of the two-stage detector significantly simpler than its counterpart for any other linear first stage.

If the $i$th user is linearly independent, it is clear from the argument in Section V-C that in order to show that the additive noise and the residual interference are independent, we need only to prove the following lemma.

**Lemma:** If the $i$th user is linearly independent, then for any $V \in C(H)$, $(VH)_i = \delta_{i1}$ for $i = 1, 2, \ldots, K$.

**Proof:** Invoking Lemma 1 in [8], we have that if the $i$th user is linearly independent, $[H^iH]_{ij} = \delta_{ij}$ for all $j$ and for any $H^i \in I(H)$. From the definition of $C(H)$, it is clear that

$$ (VH)_i = \delta_{i1} \forall j \text{ and } \forall i \in I $$

(A.1)

$\delta_{ij}$ denotes the Kronecker delta.
where \( I \) denotes the set of indexes corresponding to users that are linearly independent. Let \( \bar{I} \) denote the set of indexes corresponding to linearly dependent users. Now for \( k \in \bar{I} \), consider the equalities

\[
(VH)_{kj} = (H^+H)_{kj} = (H^+H)_{jk}, \quad \forall \ j
\]

where the first equality follows from the definition of \( V \) in (4.16) and the second equality follows from the definition of the Moore–Penrose generalized inverse. For \( j \in I \), using the above referenced lemma and noting that \( j \) cannot be equal to \( k \), we have \((H^+H)_{jk} = \delta_{jk} = 0\). Considering that (A.2) is valid for all \( j \), this implies that

\[
(VH)_{kj} = 0 \quad \forall \ j \in I \quad \text{and} \quad \forall \ k \in \bar{I}. \tag{A.3}
\]

Therefore for any \( i \in I \), we have from (A.1) and (A.3) that

\[
(VH)_{il} = \delta_{il} \quad \forall \ l \in I \cup \bar{I} . \tag{A.4}
\]

and hence the sought result. It should be pointed out here that an analogous result holds for the class of linear detectors given by \( E^{1/2} SE^{-1/2} \) where \( S \in \mathcal{C}(R) \).

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